

# Irrelevance of $f_0(500)$ in bulk thermal properties \*

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We discuss why the scalar-isoscalar resonance  $f_0(500)$  should in practice not be included in thermal models describing the freeze-out of heavy-ion collisions. Its contribution into pion multiplicities is in principle relevant because it is light and it decays only into pions. However, it is cancelled to a very good numerical precision by the non-resonant scalar-isotensor repulsion among pions. Our approach is an application of a well-known theorem relating spectral function to phase shifts. The numerical results are solely based on pion-pion scattering data and thus model independent.

## 1. Introduction

The scalar-isoscalar resonance  $f_0(500)$  is now firmly established [1]. The Particle Data Group (PDG) reports the position of its pole in the range  $(400 - 550) - i(200-350)$  [2]. Investigations based on dispersive analysis show even smaller errors:  $(400 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$  in Ref. [3] and  $(457^{+14}_{-13}) - i(279^{+11}_{-7})$  in Ref. [4].

The resonance  $f_0(500)$  is the lightest scalar state; moreover, it decays only into pions. Then, one is lead to think that  $f_0(500)$  is important for the determination of pion multiplicities in thermal models for relativistic heavy-ion collisions (see e.g. Refs. [6, 7] and refs. therein). Indeed, a simple inclusion of  $f_0(500)$  as a Breit-Wigner resonance would lead to a sizable increase (about 3-5% [8]) of pions. However, in these proceedings (based on the findings of Ref. [5]), we show that this conclusion is not correct. In fact, when the repulsion of pion-pion interaction in the scalar-isotensor

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channel is properly taken into account using the formalism developed in Refs. [9, 10, 11], the effect of  $f_0(500)$  cancels to a very good numerical accuracy. We show this cancellation in a model independent way, since the only input is given by well-known pion-pion scattering data in these two scalar channels.

As a net result, one can neglect in thermal models both the scalar-isoscalar attraction due to  $f_0(500)$  and the non-resonant scalar-isotensor repulsion.

## 2. Cancellation of $f_0(500)$

A successful description of hadron emissions at the freeze-out of relativistic heavy-ion collisions is achieved with the help of thermal models. For simplicity, we restrict our presentation to a gas which includes stable pions ( $I = 0$ ,  $J^{PC} = 0^{-+}$ , where  $I$  stays for isospin,  $J$  for the total spin, and  $P$  and  $C$  for parity and charge-conjugation), the  $\rho$ -resonance ( $I = 1$ ,  $J^{PC} = 1^{--}$ ), the resonance  $f_0(500)$  ( $I = 0$ ,  $J^{PC} = 0^{++}$ ), and the non-resonant contribution of the repulsion in the  $I = 2$ ,  $J^{PC} = 0^{++}$  channel. (Other contributions with different  $I$  and  $J^{PC}$  correspond to heavier mesons and are neglected here.)

The logarithm of the partition function  $Z$  is given by the sum of contributions of all channels:

$$\ln Z = \ln Z_\pi + \ln Z_{(1,1^{--})} + \ln Z_{(0,0^{++})} + \ln Z_{(2,0^{++})} .$$

All other thermodynamic quantities follow:  $P = -T \ln Z/V$ ,  $\varepsilon = -\partial_\beta \ln Z/V$ , etc. For what concerns stable pions (we do not include chemical potentials), one has

$$\ln Z_\pi = 3V \int_p \ln \left[ 1 - e^{-\frac{\sqrt{\vec{p}^2 + M_\pi^2}}{T}} \right]^{-1} , \quad \int_p = \int \frac{d^3 p}{(2\pi)^3} ,$$

where  $V$  is the volume,  $\vec{p}$  the pion momentum,  $M_\pi$  the pion mass, and the factor 3 the isospin degeneracy. Following Refs. [9, 10] we can express the contribution in the channel  $I = 1$ ,  $J^{PC} = 1^{--}$  as:

$$\ln Z_{(1,1^{--})} = 3 \cdot 3 \int_0^{\Lambda_1} dm \frac{d\delta_{(1,1)}(m)}{\pi dm} \int_p \ln \left[ 1 - e^{-\frac{\sqrt{\vec{p}^2 + m^2}}{T}} \right]^{-1} , \quad (1)$$

where  $\delta_{(1,1)}$  is the measured  $\pi\pi$  phase-shift as function of  $m = \sqrt{s}$ . We set  $\Lambda_1 = 1$  GeV as maximal energy in the integral, then only the  $\rho$ -meson is present in this range. The spectral function of the  $\rho$ -meson is related to the phase-shift by [9, 10]:

$$d_\rho(m) = \frac{d\delta_{(1,1)}(m)}{\pi dm} . \quad (2)$$

Thus, one can take into account the  $\rho$ -meson in a thermal gas in a model independent way by introducing the well-known scattering data in Eq. (1). For a small width  $d_\rho(m)$  can be well approximated by a Breit-Wigner function,  $d_\rho(m) \simeq \frac{\Gamma}{2\pi} [(m - M_\rho)^2 + \Gamma^2/4]^{-1}$ , and, in the limit of zero width, one correctly obtains  $d_\rho(m) = \delta(m - M_\rho)$ . Thus, the example of the  $\rho$ -meson shows quite general features of a thermal gas. The approximation of using a Breit-Wigner expression -typically used in practice- emerges.

We now turn to the main topic of the present work. For  $I = J = 0$ , the contribution of  $f_0(500)$  is included in the integral

$$\ln Z_{(0,0++)} = \int_0^{\Lambda_0} dm \frac{d\delta_{(0,0)}}{\pi dm} \int_p \ln \left[ 1 - e^{-\frac{\sqrt{p^2+m^2}}{T}} \right]^{-1} \quad (3)$$

where  $\Lambda_0 \simeq 0.8$  GeV (far above the average mass of  $f_0(500)$  but below  $f_0(980)$ , which is not considered here). The spectral function of the  $f_0(500)$  is given by  $d_{f_0(500)}(m) = \frac{1}{\pi} d\delta_{(0,0)}/dm$ . The form of  $d_{f_0(500)}(m)$  is far from being a Breit-Wigner [5] and is even not normalized to unity. This is in agreement with the fact that the resonances  $f_0(500)$  is not the chiral partner of the pion and is not a quark-antiquark field [1] (the chiral partner of  $\pi$ , the ‘true’  $\sigma$  of linear Sigma Models, should be identified with the heavier scalar resonance  $f_0(1370)$  [12]).

As a last step, we consider the joint contribution of both  $I = 0$  and  $I = 2$  channels:

$$\ln Z_{(0,0++)} + \ln Z_{(2,0++)} = \int_0^{\Lambda_0} dm \left[ \frac{d\delta_{(0,0)}}{\pi dm} + 5 \frac{d\delta_{(2,0)}}{\pi dm} \right] \int_p \ln \left[ 1 - e^{-\frac{\sqrt{p^2+m^2}}{T}} \right]^{-1} \quad (4)$$

where the factor 5 in front of  $d\delta_{(2,0)}/dm$  is the degeneracy  $2I + 1$ . Data on pion-pion scattering show the following peculiar fact [5]:

$$\frac{d\delta_{(0,0)}}{\pi dm} + 5 \frac{d\delta_{(2,0)}}{\pi dm} \simeq 0 \text{ for } 2M_\pi \leq m \lesssim 0.8 \text{ GeV}, \quad (5)$$

which is valid to a very good numerical accuracy. Then  $\ln Z_{(0,0++)} + \ln Z_{(2,0++)} \simeq 0$ ! The contribution of  $f_0(500)$  cancels.

### 3. Conclusions

In these proceedings we have shown that the resonance  $f_0(500)$  can in practice be neglected in all isospin-averaged quantities of a thermal hadronic gas, e.g. for pion multiplicities. Then, the proton-to-pion puzzle becomes even stronger, leaving other explanations open [13]. On the other hand, in correlation studies of pion-pair production the cancellation does not occur, hence  $f_0(500)$  may play a relevant role [14].

A similar cancellation occurs for the not yet confirmed  $K_0^*(800)$  ( $I = 1/2, J^{PC} = 0^{++}$ , e.g. Ref. [15] and refs. therein), whose contribution is (only partly) compensated by the  $I = 3/2, J^{PC} = 0^{++}$  channel [5, 16].

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